

Trellis-Extended Codebooks and Successive Phase Adjustment: A Path from LTE-Advanced to FDD Massive MIMO Systems

Junil Choi, David J. Love, and Taeyoung Kim

Abstract—It is of great interest to develop efficient ways to acquire accurate channel state information (CSI) for frequency division duplexing (FDD) massive multiple-input multiple-output (MIMO) systems for backward compatibility. It is theoretically well known that the codebook size for CSI quantization should be increased as the number of transmit antennas becomes larger, and 3GPP long term evolution (LTE) and LTE-Advanced codebooks also follow this trend. Thus, in massive MIMO, it is hard to apply the conventional approach of using pre-defined vector-quantized codebooks for CSI quantization mainly because of codeword search complexity. In this paper, we propose a trellis-extended codebook (TEC) that can be easily harmonized with current wireless standards such as LTE or LTE-Advanced by extending standardized codebooks designed for 2, 4, or 8 antennas with trellis structures. TEC exploits a Viterbi decoder and convolutional encoder in channel coding as the CSI quantizer and the CSI reconstructor, respectively. By quantizing multiple channel entries simultaneously using standardized codebooks in a state transition of trellis search, TEC can achieve fractional bits per channel entry quantization to have a practical feedback overhead. Thus, TEC can solve both the complexity and the feedback overhead issues of CSI quantization in massive MIMO systems. We also develop trellis-extended successive phase adjustment (TE-SPA) which works as a differential codebook of TEC. This is similar to the dual codebook concept of LTE-Advanced. TE-SPA can reduce CSI quantization error even with lower feedback overhead in temporally correlated channels. Numerical results verify the effectiveness of the proposed schemes in FDD massive MIMO systems.

Index Terms—Massive MIMO, limited feedback, trellis-extended codebook, trellis-extended successive phase adjustment.

I. INTRODUCTION

The 3GPP LTE-Advanced standard has been recently finalized during the last few years [1], and commercial products that support LTE-Advanced are about to be released in the market. LTE-Advanced is able to deploy up to eight antennas at the base station meaning that efficient downlink/uplink multiple-input multiple-output (MIMO) techniques can be exploited [2]. To quantize channel state information (CSI) more efficiently, LTE-Advanced introduced dual codebooks that quantize long-term/wideband and short-term/subband CSI separately [3]. However, LTE-Advanced still relies on pre-

defined vector-quantized codebooks with a codeword search complexity that grows exponentially with the codebook size.

Recently, the concept of using a very large number of antennas at the base station, known as massive MIMO or large-scale MIMO, is drawing considerable interest from both academia and industry to further enhance the total network throughput by implementing aggressive multi-user MIMO (MU-MIMO) systems [4], [5]. To exploit the full benefit of massive MIMO, the base station needs to have accurate CSI of the channel between the base station and users. Thus, the challenge is to scale channel estimation and feedback methods to provide CSI effectively. Most of the literature on massive MIMO focuses on time division duplexing (TDD) to sidestep this challenge. By relying on TDD, CSI can be extracted implicitly by using uplink pilot signals and the downlink/uplink channel reciprocity property assuming the transmit and receive antennas RF chains are properly calibrated [6]. However, frequency division duplexing (FDD) dominates most of the cellular market today, and it is expected that FDD would be adopted for at least the first stage of massive MIMO deployments when the number of transmit antennas is *not very large*, e.g., 32 or 64 antennas, for backward compatibility [5]. Thus, it is of great interest to develop efficient ways to acquire accurate CSI for FDD massive MIMO systems.

To implement FDD massive MIMO systems, we need to develop 1) a novel training technique for downlink channel estimation and 2) an efficient CSI quantization method. Note that the overhead of downlink training (relying on conventional unitary training techniques) and CSI quantization must both scale proportional to the number of transmit antennas to have accurate channel estimation at the user and to maintain a certain level of CSI quantization loss [7], [8]. Because of the very large number of antennas, the overhead for both unitary training and vector-quantized codebook based CSI feedback might overwhelm whole downlink and uplink resources in massive MIMO systems. Moreover, the complexity of CSI quantization using vector-quantized codebooks increases exponentially with the feedback overhead (or the number of transmit antennas). These heavy training/feedback overhead and the CSI quantization complexity problem should be solved to implement practical FDD massive MIMO systems.

Recently, some works have been dedicated to solving these issues. In [9], [10], efficient training techniques of which the downlink training overhead does not increase linearly with the number of transmit antennas are proposed. For the CSI quantization issue, a compressed sensing based CSI quantization

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approach is proposed in [11], and a overall feedback reduction technique that exploits spatial correlation of users is proposed in [12].

In this paper, we propose a trellis-extended codebook (TEC) for FDD massive MIMO systems. TEC combines a trellis encoder (i.e., using Viterbi decoding) and a vector-quantized codebook to quantize a large dimension of channels. Similar CSI quantization approaches that exploit the trellis encoder have been proposed in [13], [14]. In [13], the trellis search and vector-quantized codebooks are both considered as in TEC. However, the path metric for the trellis search is hand-optimized in [13], resulting in severe performance degradation. On the other hand, the duality between noncoherent sequence detection and CSI quantization is exploited in [14] to set an appropriate path metric for the trellis search. However, [14] relies on standard constellation points such as phase shift keying (PSK) or quadrature amplitude modulation (QAM) to quantize a channel vector, which gives a large feedback overhead and does not lead to a straightforward extension of existing 3GPP codebooks.

The proposed TEC adopts the same path metric as in [14]; however, TEC utilizes vector-quantized codebooks rather than constellation points. Therefore, TEC can easily satisfy backward compatibility by exploiting standardized LTE or LTE-Advanced codebooks and achieve a fractional bits per channel entry quantization to have practical feedback overhead. TEC also can utilize other codebooks, e.g., a Grassmannian line packing (GLP) codebook [15], [16], a random vector quantized (RVQ) codebook [8], [17]. We develop a codewords-to-trellis branches mapping rule to maximize the performance of TEC. We also investigate a codebook design methodology (instead of reusing conventional codebooks) that is suitable to TEC in this paper.

Moreover, we propose trellis-extended successive phase adjustment (TE-SPA) which functions as a differential version of TEC. Note that differential codebooks exploit temporal correlation of channels to reduce quantization error in a successive manner [18]–[25]. TE-SPA also quantizes channels successively in time and can reduce quantization loss even with reduced feedback overhead than TEC. The concept of TEC and TE-SPA is similar to LTE-Advanced dual codebooks, i.e., TEC quantizes long-term/wideband CSI while TE-SPA quantizes short-term/subband CSI.

The remainder of this paper is organized as follows. We explain the system model we consider in Section II. The proposed TEC and TE-SPA are explained in Section III and Section IV, respectively. Simulation results are presented in Section V, and conclusions follow in Section VI.

II. SYSTEM MODEL

For simple explanation, we first consider a block fading multiple-input single-output (MISO) channel with M_t transmit antennas at the base station and single receive antenna at the user as shown in Fig. 1. The proposed TEC can be easily extended to a multiple receive antenna case as explained in Section III-C. The received signal for channel use index ℓ in

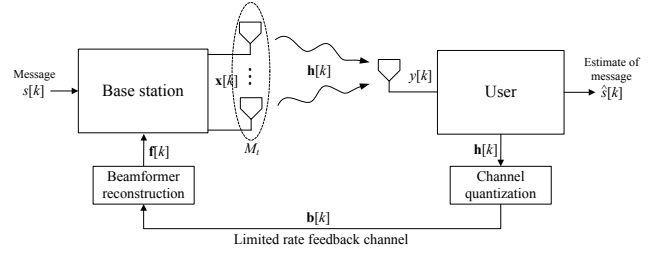


Fig. 1: Multiple-input, single-output communications system with limited feedback.

the k th fading block, $y_\ell[k] \in \mathbb{C}$, is written as

$$y_\ell[k] = \sqrt{P} \mathbf{h}^H[k] \mathbf{f}[k] s_\ell[k] + z_\ell[k],$$

where P is the transmit power, $\mathbf{h}[k] \in \mathbb{C}^{M_t}$ is the MISO channel vector, $\mathbf{f}[k] \in \mathbb{C}^{M_t}$ is the unit norm beamforming vector, $s_\ell[k] \in \mathbb{C}$ is the message signal satisfying $E[s_\ell[k]] = 0$ and $E[|s_\ell[k]|^2] = 1$, and $z_\ell[k] \sim \mathcal{CN}(0, \sigma^2)$ is complex additive white Gaussian noise.

For CSI quantization, we assume that the total number of feedback bits B_{tot} scales linearly with M_t as

$$B_{tot} \triangleq B M_t$$

where B is the number of quantization bits per transmit antenna. The linear increment of the feedback overhead is necessary to achieve a certain level of channel quantization error [8] or a full multiplexing gain of MU-MIMO [26].

If we rely on the conventional approach of using B_{tot} -bit unstructured vector-quantized codebook $\mathcal{C} = \{\mathbf{c}_1, \dots, \mathbf{c}_{2^{B_{tot}}}\}$ consists of unit norm codewords for CSI quantization, the user quantizes its channel by selecting the best codeword \mathbf{c}_{opt} that aligns with the channel most closely as

$$\mathbf{c}_{opt}[k] = \underset{\mathbf{c} \in \mathcal{C}}{\operatorname{argmax}} |\mathbf{h}^H[k] \mathbf{c}|^2, \quad (1)$$

and feeds back the binary index of $\mathbf{c}_{opt}[k]$, i.e., $\mathbf{b}[k] = \text{bin}(\text{opt})$ where $\text{bin}(\cdot)$ converts an integer to its binary representation, to the base station.

Note that the codeword search complexity of using vector-quantized codebooks is given as $O(M_t 2^{B_{tot}})$. If B_{tot} or M_t is small as in current cellular systems, the complexity of CSI quantization is not a problem. However, in massive MIMO systems with a very large number of M_t , brute force codeword selection becomes infeasible.

III. TRELLIS-EXTENDED CODEBOOK (TEC)

TEC can exploit and extend pre-existing vector-quantized codebooks such as LTE or LTE-Advanced codebooks. Because of its backward compatibility, TEC is an excellent candidate for CSI quantization of future FDD massive MIMO systems. We first explain the exact procedure of TEC. We then discuss codewords-to-trellis branches mapping and codebook design criteria to maximize the performance of TEC. Because we do not consider temporal correlation of channels in this section, we drop the time index k to simplify notations for the remainder of this section.

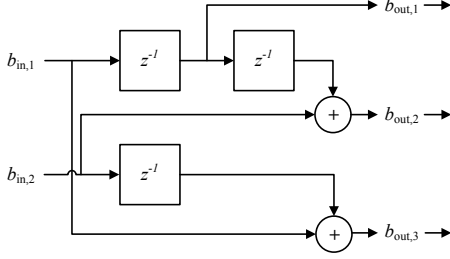


Fig. 2: A rate $\frac{2}{3}$ convolutional encoder that can be used to generate a TEC codebook. In the figure, $b_{in,1}$ and $b_{in,2}$ are the least significant and the most significant input bits, respectively. Same for the output bits.

A. TEC Procedure

TEC is based on the equivalence between the two optimization problems

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{C}^N}{\operatorname{argmin}} \min_{\theta \in [0, 2\pi)} \|\mathbf{y} - e^{j\theta} \mathbf{x}\|_2^2$$

and

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{C}^N}{\operatorname{argmax}} \frac{|\mathbf{y}^H \mathbf{x}|^2}{\|\mathbf{x}\|_2^2}. \quad (2)$$

Note that (2) is the same as (1) when $\|\mathbf{x}\|_2^2 = 1$. Thus, we can transform the CSI quantization problem in (1) to

$$\mathbf{c}_{\text{opt}} = \underset{\mathbf{c} \in \mathcal{C}}{\operatorname{argmin}} \min_{\theta \in [0, 2\pi)} \|\mathbf{h} - e^{j\theta} \mathbf{c}\|_2^2. \quad (3)$$

Instead of searching θ over the continuous space $[0, 2\pi)$, we can discretize the search space, i.e., $\theta \in \Theta = \{\theta_1, \dots, \theta_{K_\theta}\}$, as in noncoherent sequence detection [27]. With a given θ , (3) can be efficiently solved by well-known source coding techniques such as trellis coded modulation (TCQ) or trellis quantizer [28]. This conversion is successfully exploited in [14], [29] to develop efficient CSI quantizers. TEC also solves (3) using trellis quantizers similar to [14]. The main difference is that [14] handles one channel entry per state transition of the trellis search while TEC processes multiple channel entries simultaneously.

TEC can be implemented with any trellis quantizer. In this paper, we adopt the Ungerboeck trellis and convolutional encoder [30] because of their simplicity and good performance. Let B_{in} and B_{out} be the number of input and output bits of a convolutional encoder of interest, respectively. The Ungerboeck convolutional encoder satisfies $B_{out} = B_{in} + 1$. Note that the trellis of the corresponding convolutional encoder has $2^{B_{out}}$ states, and each state has $2^{B_{in}}$ branches. However, the total number of distinctive branches is $2^{B_{out}}$. One example of the rate $\frac{2}{3}$ convolutional encoder from [30] and the corresponding trellis are shown in Fig. 2 and 3, respectively. As shown in Fig. 3, each state has four branches differentiated with inputs and even or odd outputs.

Let L denote the number of simultaneously quantized channel elements in a state transition of trellis. We assume that L divides the number of transmit antennas M_t . Note that TEC supports $B = \frac{B_{in}}{L}$ bits per channel entry quantization,

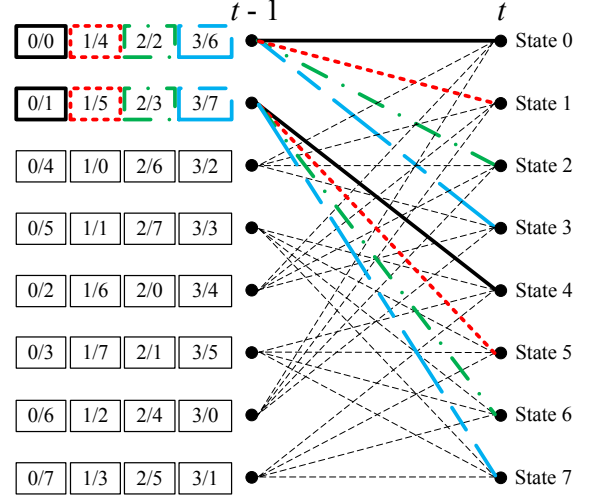


Fig. 3: The trellis representation of the convolutional encoder in Fig. 2. Each state transition in the right side is mapped with input/output relation using decimal numbers in each box in the left. For example, 1/4 (in decimal numbers) in the top red-dot box represents the state transition from the state 0 to the state 1 with input=01/output=100 (all in binary numbers).

which will become clear later. Thus, if $L > B_{in}$, TEC can achieve fractional bits per channel entry quantization.

To process L channel entries per state transition, TEC maps branches in the trellis with $L \times 1$ codewords $\mathbf{c}_k^L \in \mathbb{C}^L$. To do this, we need to have a vector-quantized codebook (such as the LTE codebook) with $2^{B_{out}}$ codewords, i.e., $\mathcal{C}_{2^{B_{out}}}^L = \{\mathbf{c}_1^L, \dots, \mathbf{c}_{2^{B_{out}}}^L\}$, to map all $2^{B_{out}}$ branches of trellis with different outputs. We will discuss the codewords-to-trellis branches (or outputs) mapping and the codebook design criteria later. For the time being, we assume that all $2^{B_{out}}$ branches are mapped with some codewords.

To perform the trellis search using the Viterbi algorithm, we need to define a path metric which can solve (3) on the fly. Let \mathbf{p}_t be a partial path up to the stage t in the trellis. We also define $\text{in}(\mathbf{p}_t)$ as the binary input sequence corresponding to path \mathbf{p}_t and $\text{out}(\mathbf{p}_t)$ as the sequence of codewords \mathbf{c}_k^L 's that are mapped with branches in the path \mathbf{p}_t . Note that $\text{out}(\mathbf{p}_t) \in \mathbb{C}^{Lt}$ where each block of L entries of $\text{out}(\mathbf{p}_t)$ is from a specific codeword \mathbf{c}_k^L . With these definitions, we can define the path metric based on (3) as

$$\begin{aligned} m(\mathbf{p}_t, \theta) &= \|\mathbf{h}_{[1:Lt]} - e^{j\theta} \text{out}(\mathbf{p}_t)\|_2^2 \\ &= m(\mathbf{p}_{t-1}, \theta) + \|\mathbf{h}_{[L(t-1)+1:Lt]} - e^{j\theta} \text{out}([p_{t-1} \ p_t])\|_2^2 \end{aligned} \quad (4)$$

where $\mathbf{h}_{[m:n]}$ is the truncated vector of \mathbf{h} from the m th entry to the n th entry. The path metric in (4) can be efficiently computed with a given candidate value of θ using the Viterbi algorithm where the total number of stages in the trellis is equal to $T = \frac{M_t}{L}$. The best path \mathbf{p}_{best} and the best phase θ_{best} that minimize the path metric in (4) is given by solving

$$\min_{\theta \in \Theta} \min_{\mathbf{p}_T \in \mathbb{P}_T} m(\mathbf{p}_T, \theta)$$

where \mathbb{P}_T denotes all possible paths up to stage T . The best codeword \mathbf{c}_{opt} and the binary feedback sequence \mathbf{b} are given as

$$\mathbf{c}_{\text{opt}} = \text{out}(\mathbf{p}_{\text{best}}), \quad \mathbf{b} = \text{in}(\mathbf{p}_{\text{best}}),$$

respectively. If we normalize \mathbf{c}_k^L as $\|\mathbf{c}_k^L\|_2^2 = \frac{L}{M_t}$ for all k , then we have $\|\mathbf{c}_{\text{opt}}\|_2^2 = 1$. It is important to point out that \mathbf{b} consists of input bits (not output bits) of the convolutional encoder, results in $B = \frac{B_{\text{in}}}{L}$ bits per channel entry quantization.

Note that searching over θ only increases complexity, not the feedback overhead of TEC. The base station only needs to know the binary feedback sequence \mathbf{b} that represents the best path \mathbf{p}_{best} to reconstruct \mathbf{c}_{opt} using the convolutional encoder. We fix the starting state of trellis search to the first state not to have any additional feedback overhead in TEC.

B. Codewords-to-Trellis Branches Mapping and Codebook Design Criteria for TEC

To exploit a pre-existing vector quantized codebook in TEC, we need a clever mapping rule between codewords in $\mathcal{C}_{2^{B_{\text{tot}}}}^L$ and branches in the trellis. The mapping rule should depend on the structure of the given trellis or convolutional encoder. We propose a mapping rule for the trellis structure in Fig. 3 with an arbitrary codebook $\mathcal{C}_{2^{B_{\text{tot}}}}^L$. Similar mapping rules can be defined for other trellis structures.

1) Codewords-to-trellis branches mapping rule for Fig. 3:

Because we fix the starting state of trellis search as the first state in TEC, we only need to consider the distinctive pairs of paths in Fig. 4. Considering the red-solid paths and the first state transition of the blue-dot paths, we can conclude that we need to maximize the minimum Euclidean distance between codeword pairs that are mapped to all even outputs. For odd outputs; however, we need to separately maximize the Euclidean distance between the two codewords that are mapped to outputs $\{1, 5\}$ and $\{3, 7\}$.

To realize this, with some abuse of notation, let \mathcal{C}_1^L and \mathcal{C}_2^L denote all possible partitions of $\mathcal{C}_{2^{B_{\text{tot}}}}^L$ satisfying

$$\begin{aligned} \mathcal{C}_1^L \cup \mathcal{C}_2^L &= \mathcal{C}_{2^{B_{\text{tot}}}}^L, \\ \mathcal{C}_1^L \cap \mathcal{C}_2^L &= \emptyset, \\ \text{card}(\mathcal{C}_1^L) &= \text{card}(\mathcal{C}_2^L) = 2^{B_{\text{tot}}-1} \end{aligned}$$

where $\text{card}(\cdot)$ is the cardinality of an associated set, and \emptyset denotes an empty set. Let $\mathbf{c}_{m,k} \in \mathcal{C}_k^L$ for $k = 1, 2$. We denote $\mathcal{C}_{\text{odd}}^L$ and $\mathcal{C}_{\text{even}}^L$ as the set of codewords mapped to the trellis branches of odd and even outputs, respectively. We generate $\mathcal{C}_{\text{odd}}^L$ and $\mathcal{C}_{\text{even}}^L$ as

$$\begin{aligned} \mathcal{C}_{\text{odd}}^L &= \arg\max_{\mathcal{C}_1^L \subset \mathcal{C}^L} \min_{m \neq n} \|\mathbf{c}_{m,1} - \mathbf{c}_{n,1}\|_2^2, \\ \mathcal{C}_{\text{even}}^L &= \arg\max_{\mathcal{C}_2^L \subset \mathcal{C}^L} \min_{m \neq n} \|\mathbf{c}_{m,2} - \mathbf{c}_{n,2}\|_2^2, \end{aligned} \quad (5)$$

respectively. Once we have $\mathcal{C}_{\text{odd}}^L$ and $\mathcal{C}_{\text{even}}^L$ as above, we can have arbitrary mappings between the codewords in $\mathcal{C}_{\text{even}}^L$ and the trellis branches of even outputs. For the trellis branches of odd outputs; however, we need one more step. We divide

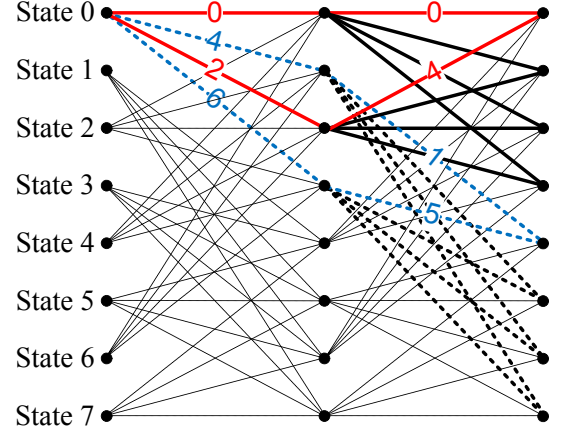


Fig. 4: Distinctive pairs of paths of which the Euclidean distance should be maximized. Two pairs of paths are highlighted with trellis outputs.

$\mathcal{C}_{\text{odd}}^L$ into $\mathcal{C}_{\text{odd},1}^L$ and $\mathcal{C}_{\text{odd},2}^L$ as we divide $\mathcal{C}_{2^{B_{\text{tot}}}}^L$ into $\mathcal{C}_{\text{odd}}^L$ and $\mathcal{C}_{\text{even}}^L$. Then, we map the codewords in $\mathcal{C}_{\text{odd},k}^L$ to the trellis branches with outputs $\{(2k-1), (2k+3)\}$ for $k = 1, 2$.

2) Codebook design criterion:

Instead of reusing conventional codebooks, we can also design a codebook that is optimized for TEC. Note that the second term of the path metric in (4) is the quantization problem in the Euclidean space. Thus, we can generate the codebook with $2^{B_{\text{out}}}$ codewords of dimension $L \times 1$ that maximize the minimum Euclidean distance between all possible codeword pairs as

$$\mathcal{C}_{\text{ED}, 2^{B_{\text{out}}}}^L = \arg\max_{\mathcal{C} \in \mathcal{U}_L^{2^{B_{\text{out}}}}} d_{\text{ED}, \min}^2(\mathcal{C}) \quad (6)$$

where $\mathcal{U}_L^N \in \mathbb{C}^{L \times N}$ is the set of all $L \times N$ complex matrices with unit norm columns and

$$d_{\text{ED}, \min}^2(\mathcal{C}) \triangleq \min_{1 \leq k < l \leq 2^N} \|\mathbf{c}_k - \mathbf{c}_l\|_2^2$$

with $\mathbf{c}_k, \mathbf{c}_l \in \mathcal{C}$.

The proposed codebook design criterion exploits the same concept as the GLP codebook that maximizes the minimum chordal distance between all codeword pairs [15], [16]. The difference is that the GLP codebook directly quantizes a channel in the Grassmann manifold while the proposed codebook works in the Euclidean space.

Remark: A similar codebook design and codewords-to-trellis mapping criteria have been proposed in [13]. However, [13] first generates the $L \times 1$ Euclidean codebook with $2^{B_{\text{in}}}$ codewords (not $2^{B_{\text{out}}}$ codewords as in the proposed scheme) that are mapped to odd (or even) outputs. With some abuse of notation, denote this Euclidean codebook $\mathcal{C}_{\text{odd}}^L$. Then $\mathcal{C}_{\text{even}}^L$ is generated by rotating $\mathcal{C}_{\text{odd}}^L$ with a unitary matrix \mathbf{U} where \mathbf{U} is designed to maximize the minimum chordal distance between codewords in $\mathcal{C}_{\text{odd}}^L \cup \mathcal{C}_{\text{even}}^L$. Because \mathbf{U} tries to maximize the minimum chordal distance, not the minimum Euclidean distance, the approach in [13] cannot guarantee to maximize the minimum Euclidean distance between all possible pairs of

codewords generated by TEC. Moreover, [13] cannot utilize an existing vector-quantized codebook different from TEC.

C. TEC for Multiple Receive Antennas

We can easily modify the proposed TEC to accommodate MIMO with M_r receive antennas at the user. Assume that $M_t \geq M_r$ and the base station transmits $K \leq M_r$ data streams simultaneously. Then, we need to quantize the first K dominant eigenvectors of $\mathbf{H}\mathbf{H}^H$, which is denoted as $\mathbf{U}(\mathbf{H}) \in \mathbb{C}^{M_t \times K}$. We can rewrite the path metric defined in (4) as

$$\begin{aligned} m(\mathbf{p}_t, \theta) &= \left\| \mathbf{U}(\mathbf{H})_{[1:Lt]} - e^{j\theta} \text{out}(\mathbf{p}_t) \right\|_F^2 \\ &= m(\mathbf{p}_{t-1}, \theta) + \left\| \mathbf{U}(\mathbf{H})_{[L(t-1)+1:Lt]} - e^{j\theta} \text{out}([p_{t-1} \ p_t]) \right\|_F^2 \end{aligned}$$

where $\mathbf{A}_{[m:n]}$ is the truncated matrix of \mathbf{A} from the m th row to the n th row, and $\|\mathbf{A}\|_F$ denotes the Frobenius norm of a matrix \mathbf{A} .

For the multiple receive antenna case, instead of using vector codewords \mathbf{c}_k^L , we need to use matrix codewords $\mathbf{C}_k^{L \times K} \in \mathbb{C}^{L \times K}$ to quantize $\mathbf{U}(\mathbf{H})$. We can use the same codebook design and codewords-to-trellis branches mapping criteria to the multiple receive antenna case by changing the 2-norm operation to a Frobenius norm operation.

IV. TRELLIS-EXTENDED SUCCESSIVE PHASE ADJUSTMENT (TE-SPA)

In practice, channels are correlated in time. There has been much work on differential codebooks that leverage the temporal correlation of channels for better CSI quantization quality, e.g., [18]–[25]. However, most of those works focused on a small number of transmit antennas and feedback bits. Thus, we propose TE-SPA which is a differential codebook version of TEC for massive MIMO systems.

We consider temporally correlated channels that are modeled by a first order Gauss-Markov process as

$$\mathbf{h}[k] = \eta \mathbf{h}[k-1] + \sqrt{1-\eta^2} \mathbf{g}[k] \quad (7)$$

where $0 \leq \eta \leq 1$, $\mathbf{h}[k]$, and $\mathbf{g}[k]$ are the correlation coefficient, the channel realization at time k , and the innovation process at time k , respectively. We assume that $\mathbf{h}[0]$ is independent of $\mathbf{g}[k]$ for all k . Note that the model in (7) is also applicable to frequency correlated channels if k denotes subcarrier or subband index of a wideband channel.

If channel variation is small in time, i.e., η is close to 1, we can successively reduce quantization error by adjusting the phase of each entry or the block of entries of previous CSI. TE-SPA adjusts phases in a block-wise manner to reduce the feedback overhead. TE-SPA consists of *block-wise phase adjustment matrix generation* and *block shifting*.

1) Block-wise phase adjustment matrix generation:

Let $\hat{\mathbf{h}}_{k-1} = \mathbf{c}_{\text{opt}}[k-1]$ and $\mathbf{h}_k = \mathbf{h}[k]$ represent the previous (quantized) CSI and the current channel vector,

respectively, to simplify notations. TE-SPA quantizes the channel at time k by adjusting the phases of $\hat{\mathbf{h}}_{k-1}$ in a block-wise manner. That is, $\hat{\mathbf{h}}_{k-1}$ is rotated with a block-wise phase adjustment matrix \mathbf{R}_k which is given as¹

$$\mathbf{R}_k = \text{diag}([e^{j\varphi_{k,1}}, \dots, e^{j\varphi_{k,T}}] \otimes \mathbf{1}_L) \quad (8)$$

where $T = \frac{M_t}{L}$, \otimes is the Kronecker product, and $\mathbf{1}_L = [1, \dots, 1]^T$ is the length L all 1 vector. Then, the quantized version of the current CSI becomes

$$\hat{\mathbf{h}}_k = \mathbf{R}_k \hat{\mathbf{h}}_{k-1}.$$

TE-SPA exploits the trellis structure as in TEC to generate \mathbf{R}_k , i.e., TE-SPA selects $\varphi_{k,n}$'s from a given set $\Psi = \{\psi_1, \dots, \psi_{2^{B_{\text{out}}}}\}$ as

$$(\varphi_{k,1}, \dots, \varphi_{k,T}) = \underset{\varphi_{k,n} \in \Psi}{\text{argmin}} \min_{\theta \in \Theta} \left\| \mathbf{h}_k - e^{j\theta} \mathbf{R}_k \hat{\mathbf{h}}_{k-1} \right\|_2^2 \quad (9)$$

using the Viterbi algorithm. Note that the convolutional encoders for TEC and TE-SPA can be different, e.g., we could adopt a rate $\frac{2}{3}$ convolutional encoder for TEC while a rate $\frac{1}{2}$ convolutional encoder is used for TE-SPA to reduce successive feedback overhead.

To quantize CSI effectively, we need to appropriately set the values of the elements in Ψ and assign those elements to the trellis branches, which are exactly the same principles as the codebook design and the codeword-to-trellis branch mapping criteria in TEC. Previous works on differential codebook tried to optimize codebook update methods taking the temporal correlation coefficient η into account. In TE-SPA, this is implicitly handled during the trellis search, i.e., the trellis search selects the best set of phases for \mathbf{R}_k which rotate the previous CSI “close” to the current channel. Therefore, it is better to have the values of elements in Ψ such that they are able to generate as different rotation matrices as possible. Note that \mathbf{R}_k is determined by the relation among the $\varphi_{k,n}$'s. If $T = 2$, then $\text{diag}([1, e^{j\frac{\pi}{4}}] \otimes \mathbf{1}_L)$ is the same as $\text{diag}([e^{j\frac{\pi}{4}}, 1] \otimes \mathbf{1}_L)$ in terms of \mathbf{R}_k . Thus, we can restrict the search space to $[0, \pi]$ and assign the values in Ψ as

$$\psi_\nu = \frac{\nu-1}{2^{B_{\text{out}}}} \pi, \quad \nu = 1, \dots, 2^{B_{\text{out}}}.$$

Now, we need a mapping rule between ψ_ν 's and trellis outputs. We consider ψ_ν 's as PSK constellation points and follow the same mapping rule as in trellis coded modulation (TCM) [30]. That is, we maximize the minimum Euclidean distance among ψ_ν 's that are mapped to the branches with the same incoming/outgoing states by mapping ψ_ν to the trellis output ν .

Remark: We can further reduce the feedback overhead of TE-SPA. Note that we can rewrite \mathbf{R}_k in (8) as

$$\mathbf{R}_k = e^{j\varphi_{k,1}} \text{diag}([1, \dots, e^{j(\varphi_{k,T} - \varphi_{k,1})}] \otimes \mathbf{1}_L).$$

¹The block length L with the same phase $\varphi_{k,n}$ in \mathbf{R}_k is a design parameter and does not need to be the same as that of TEC. We assume the length of L is the same as in TEC for simple explanation.

$$(\varphi_{k,1}, \dots, \varphi_{k,T}) = \underset{\varphi_{k,n} \in \Psi}{\operatorname{argmin}} \min_{\theta \in \Theta} \left\| \mathbf{h}_k \left[\frac{L}{2}(k-1) \right]_c - e^{j\theta} \mathbf{R}_k \hat{\mathbf{h}}_{k-1} \left[\frac{L}{2}(k-1) \right]_c \right\|_2^2, \quad k \geq 1. \quad (10)$$

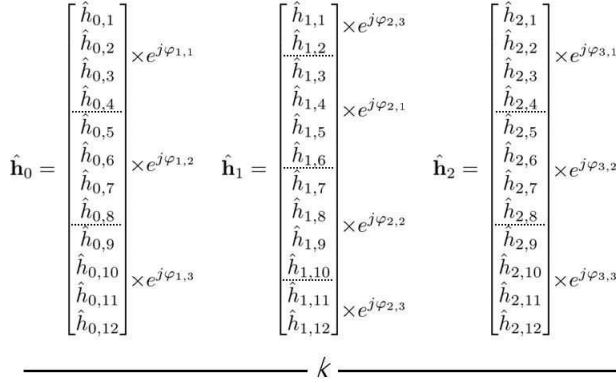


Fig. 5: A conceptual explanation of TE-SPA with block-shifting with $M_t = 12$ and $L = 4$. $\hat{\mathbf{h}}_k$ is the result of multiplying $e^{j\varphi_{k,n}}$'s to $\hat{\mathbf{h}}_{k-1}$ in a block-wise manner.

Let $\check{\mathbf{R}}_k = \text{diag}([1, \dots, e^{j(\varphi_{k,T} - \varphi_{k,1})}] \otimes \mathbf{1}_L)$. Then, the objective function in (9) can be rewritten as

$$\left\| \mathbf{h}_k - e^{j(\theta + \varphi_{k,1})} \check{\mathbf{R}}_k \hat{\mathbf{h}}_{k-1} \right\|_2^2.$$

Thus, if we appropriately redesign Θ for the noncoherent search in (9), we can always fix the first entry of $\check{\mathbf{R}}_k$ as 1 and skip (or fix) the first stage of the trellis search which gives reduced feedback overhead.

2) Block-Shifting:

If we fix the block structure of the phase adjustment matrix \mathbf{R}_k , then the performance can quickly saturate because we cannot adjust the phase relation of the elements within each block. Moreover, since we fix the starting state of the trellis search, the first state transition suffers from using a restricted number of branches, e.g., only 4 branches with even trellis outputs are exploited for the first state transition in Fig. 4. These might not be serious problems for one-shot quantization as in TEC, however, the loss could be accumulated in successive quantizations as in TE-SPA. Therefore, we adopt *block-shifting* to mitigate these problems.

Let $\mathbf{a}[m]_c$ and $\mathbf{A}[m]_c$ denote the left circularly shifting of a vector \mathbf{a} and diagonal entries of a matrix \mathbf{A} with m elements, respectively. For example, if $\mathbf{a} = [1, 2, 3, 4, 5]$, then $\mathbf{a}[2]_c = [3, 4, 5, 1, 2]$. Using this notation, we rewrite the optimization problem in (9) as in (10). We interweave two consecutive blocks by circularly shifting $\frac{L}{2}$ elements in (10) to prevent the saturation effect.² After generating \mathbf{R}_k , the quantized CSI at time k is given as

$$\hat{\mathbf{h}}_k = \mathbf{R}_k \left[-\frac{L}{2}(k-1) \right]_c \hat{\mathbf{h}}_{k-1}.$$

²To further improve performance, we can dynamically reassign the blocks of \mathbf{R}_k instead of circularly shifting elements in time.

The conceptual explanation of TE-SPA with block-shifting is shown in Fig. 5. Note that TEC is used for CSI quantization at $k = 0$. The proposed block shifting can adjust not only the phase relation among blocks but also that of elements within each block in time. Moreover, the phase $\varphi_{k,1}$ from the first state transition is multiplied to the different blocks of $\hat{\mathbf{h}}_k$ depending on k , which prevents the accumulation of the loss caused by the first state transition.

V. SIMULATIONS

We performed Monte-Carlo simulations using 10000 channel realizations to evaluate the proposed TEC and TE-SPA. We set $K_\theta = 16$ for $\Theta = \{\theta_1, \dots, \theta_{K_\theta}\}$ to perform noncoherent part of TEC and TE-SPA.

We first evaluate TEC in i.i.d. Rayleigh fading channels as $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_t})$. We use the average beamforming gain in dB scale, $10 \log_{10}(E[|\mathbf{h}^H \mathbf{c}_{\text{opt}}|^2])$, as a performance metric where the expectation is taken over \mathbf{h} . We set $L = 4$ for the dimension of codewords mapped in trellis branches. Thus, TEC schemes with $B = 3/4$ and $B = 1/2$ bits per entry quantize 4 channel elements using 3 bits and 2 bits, respectively. In Fig. 6, we plot the average beamforming gain of TEC with the proposed codewords-to-trellis branches mapping rule using different codebooks, e.g., trellis extended-Euclidean distance (TE-ED) refers to TEC using the Euclidean distance (ED) codebook defined in (6), according to the number of transmit antennas M_t . We also plot the average beamforming gain of the scheme in [14] with $B = 1$ and that of the RVQ codebook with the same feedback overhead with TEC schemes for comparison purpose. Note that TE-LTE with $B = 1/2$ refers to TEC using only the first 8 among 16 codewords of LTE 4 transmit antennas codebook, which are the same as 8 DFT codewords. The total feedback overhead of each scheme is given as $B_{\text{tot}} = BM_t$.

As expected in Section III-B, TE-ED using the ED codebook gives the best performance among TEC schemes. The gain is more than 1 dB compared to TE-LTE when $B = 3/4$ and M_t is more than 64. TE-LTE suffers from practical constraints on its codewords such as constant modulus and finite alphabet properties. The conventional vector-quantized codebook approach using the RVQ codebook is better than TEC schemes, but the plot of the RVQ codebook is based on the analytical approximation of $M_t \left(1 - 2^{-\frac{B_{\text{tot}}}{M_t-1}}\right)$ [8] because it is infeasible to simulate the performance of the RVQ codebook with $B_{\text{tot}} = 16$ bits (which is the case of $M_t = 32$ with $B = 1/2$) or more. The scheme from [14] outperforms TEC schemes but the feedback overhead is $\frac{4}{3}$ and 2 times larger than that of TEC with $B = 3/4$ and $B = 1/2$, respectively.³

³We did not compare TEC with [13] because the proposed scheme in [13] cannot even maintain a constant performance gap with the RVQ codebook.

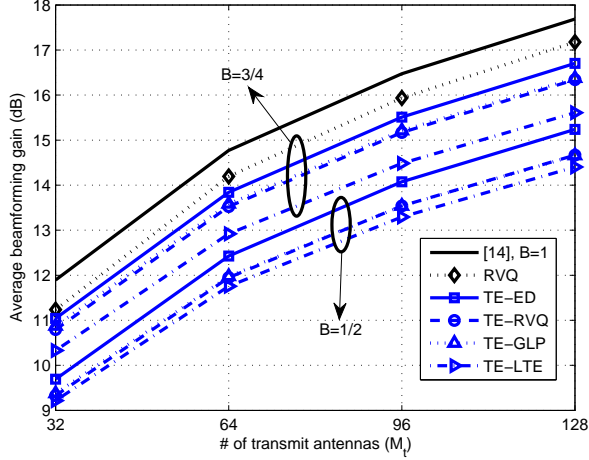


Fig. 6: Average beamforming gain (dB) with M_t in i.i.d. Rayleigh fading channels. TE-‘codebook name’ refers to TEC using the specific codebook. $B_{tot} = BM_t$.

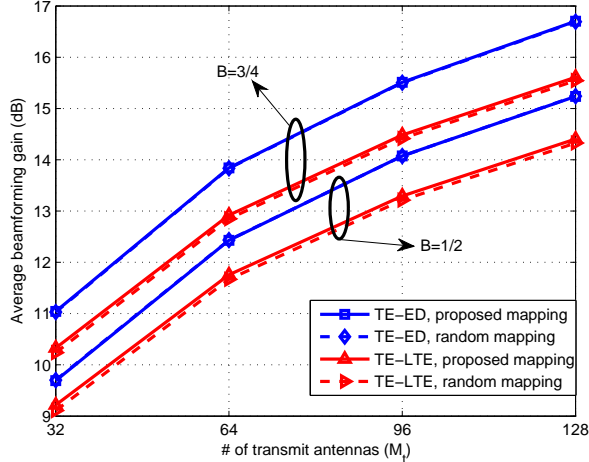


Fig. 7: Average beamforming gain (dB) with M_t in i.i.d. Rayleigh fading channels. TEC schemes with the proposed codewords-to-trellis branches mapping and random mapping are compared.

We also compare the beamforming gains of the proposed codewords-to-trellis branches mapping and a random mapping (per iteration) using TE-ED and TE-LTE in Fig. 7. Note that the proposed mapping has negligible impact on the average beamforming gain of TE-ED. The reason is that the Euclidean distance among codewords in the ED codebook is already far apart such that the random mapping is also guaranteed to have a good Euclidean distance property. On the other hand, the proposed mapping achieves around 0.1 dB gain compared to the random mapping in TE-LTE. This shows that if we reuse pre-existing vector-quantized codebooks that are not optimized in the Euclidean distance, the proposed mapping can achieve additional gain with the same codebook.

Now, we evaluate TEC for a multiple receive antenna case. We set $M_t = 16$, $M_r = 2$, and the transmission rank as

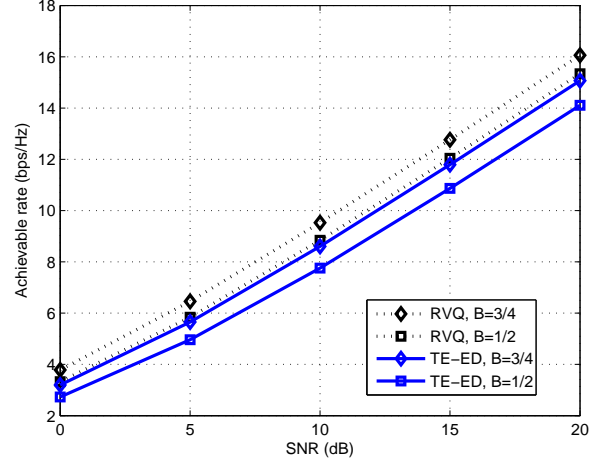


Fig. 8: Achievable rate with SNR in i.i.d. Rayleigh fading channels with $M_t = 16$, $M_r = 2$, and $K = 2$. $B_{tot} = BM_t$.

$K = 2$. The number of transmit antennas is *not so large* in this case because we want to compare TEC and the RVQ codebook with the same feedback overhead. With $M_t = 16$, TEC with $B = 3/4$ and $B = 1/2$ correspond to $B_{tot} = 12$ and $B_{tot} = 8$ bits, respectively. Denote the average achievable rate as

$$E \left[\log_2 \det \left(\mathbf{I}_K + \frac{P}{\sigma^2 K} \mathbf{F}^H \mathbf{H} \mathbf{H}^H \mathbf{F} \right) \right]$$

where $\frac{P}{\sigma^2}$ is signal-to-noise-ratio (SNR), $\mathbf{H} \in \mathbb{C}^{M_t \times M_r}$ is the channel matrix, $\mathbf{F} \in \mathbb{C}^{M_t \times K}$ is the precoder matrix, and the expectation is taken over \mathbf{H} . Each entry of \mathbf{H} is distributed with $\mathcal{CN}(0, 1)$. We plot the average achievable rates of TE-ED and the RVQ codebook in Fig. 8 with SNR. The proposed TE-ED maintains a constant gap of around 1 bps/Hz loss compared to the RVQ codebook with the same feedback overhead for all SNR values. Considering the asymptotic optimality of the RVQ codebook in high rank transmission [31], the proposed TEC can achieve a good performance even in multiple receive antenna cases with feasible complexity.

In Fig. 9, we evaluate TE-SPA with $M_t = 64$ in temporally correlated Rayleigh fading channels which is shown in (7) with $\mathbf{g}[k] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_t})$. We rely on Jake’s model for the temporal correlation coefficient [32] such that $\eta = J_0(2\pi f_D \tau)$ where $J_0(\cdot)$ is the zero-th order Bessel function, f_D is the maximum Doppler frequency, and τ is the channel instantiation interval. With practical system parameters of 2.5GHz carrier frequency, $\tau = 5ms$, and $3km/h$ user velocity, the temporal correlation coefficient is given as $\eta = 0.9881$. We do not consider any feedback delay in this simulation because it has been shown in [14] that the impact of feedback delay is marginal.

At $k = 0$, channels are quantized with TE-ED using $B = 1/2$ bits per channel entry while channels are quantized using TE-SPA using B_{SPA} bits per entry when $k \geq 1$. As shown in the figure, the average beamforming gain increases with k due to reduced quantization error using TE-SPA even with lower feedback overhead of $B_{SPA} = 1/4$. All TE-SPA schemes outperform the RVQ codebook that does not consider

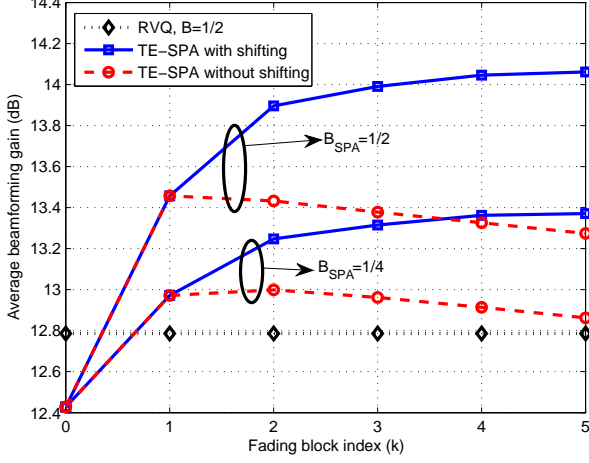


Fig. 9: Average beamforming gain (dB) with k and $M_t = 64$ in temporally correlated channels. Channels are quantized using TE-ED with $B = 1/2$ bits per entry at $k = 0$ for TE-SPA schemes. Total feedback overhead of TE-SPA at $k \geq 1$ is $B_{tot} = B_{SPA}M_t$.

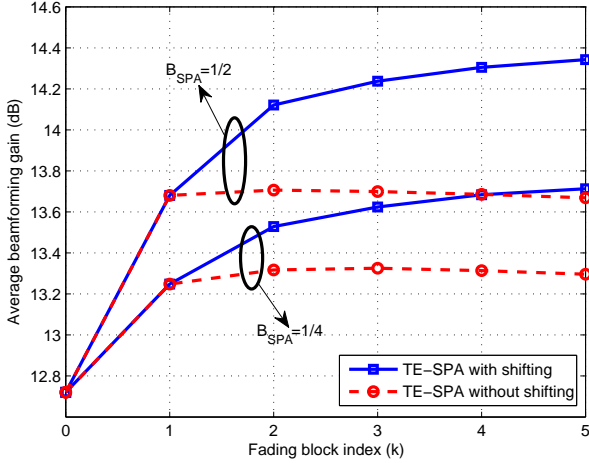


Fig. 10: Average beamforming gain (dB) with k and $M_t = 64$ using an SCM channel model. Simulation setups are the same as in Fig. 9 with uniform linear array antennas with 0.5λ antenna spacing and 8 angle spread.

temporal correlation of channels in quantization. The gain of using TE-SPA with block shifting is more than 1.6 dB when $B_{SPA} = 1/2$. Note that TE-SPA with block shifting gives far better performance than TE-SPA without block shifting because it can spread out the loss from the first state transition as explained in Section IV.

To evaluate TE-SPA in more practical scenario, we perform simulations using spatial channel model (SCM) [33] that is commonly adopted in standards such as 3GPP. In Fig. 10, we plot the average beamforming gain using the same simulation setups as in Fig. 9 with uniform linear antenna array with 0.5λ antenna spacing and 8 angle spread. As clearly shown in the figure, TE-SPA also works for the practical scenario.

VI. CONCLUSION

We proposed the trellis-extended codebook (TEC) that is an efficient channel quantization technique for FDD massive MIMO systems in this paper. The proposed TEC exploits the trellis quantizer combined with vector-quantized codebooks to achieve a practical feedback overhead and complexity. TEC can easily satisfy backward compatibility by exploiting standardized codebooks such as LTE or LTE-Advanced codebooks. We proposed the codewords-to-trellis branches mapping and the codebook design criteria to maximize the performance of TEC. TEC also can support multiple received antenna cases such that a unified CSI quantization framework is possible. It has been shown by simulations that TEC can maintain a constant performance gap with the RVQ codebook that is known to be asymptotically optimal.

We also developed trellis-extended successive phase adjustment (TE-SPA) that is a differential codebook version of TEC. We incorporated trellis structure to quantize temporally correlated channels in a successive manner. TEC and TE-SPA can be thought as a counterpart of the LTE-Advanced dual codebooks for long-term/wideband and short-term/subband CSI quantization. The numerical results confirmed that the proposed TE-SPA can reduce quantization loss even with reduced feedback overhead.

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